

# *chapter two*

## **9. Flood Routing**

Flow routing is a mathematical procedure for predicting the changing magnitude, speed, and shape of a flood wave as a function of time at one or more points along a watercourse (waterway or channel). The watercourse may be a river, reservoir, canal, drainage ditch, or storm sewer. The inflow hydrograph can result from design rainfall, reservoir release (spillway, gate, and turbine release and / or dam failures), and landslide into reservoirs.

River routing uses mathematical relations to calculate outflow from a river channel given inflow, lateral contributions, and channel characteristics are known. Channel reach refers to a specific length of river channel possessing certain translation and storage properties.

The terms river routing and flood routing are often used interchangeably. This is attributed to the fact that most stream channel-routing applications are in flood flow analysis, flood control design or flood forecasting.

Two general approaches to river routing are recognized: (1) hydrologic and (2) hydraulic. As will be discussed in Chapter 10, in the case of reservoir routing, hydrologic river routing is based on the storage concept. Hydraulic river routing is based on the principles of mass and momentum conservation. There are three types of hydraulic routing techniques: (1) kinematics wave, (2) diffusion wave, and (3) dynamic wave. The dynamic wave is the most complete model of unsteady open channel flow, kinematics and diffusion waves are convenient and practical approximations to the dynamic wave.

Hybrid model, possessing essential properties of the hydrologic routing and hydraulic routing methods are being developed. The Muskingum-Cung method is an example of a hybrid model.

This chapter discusses a commonly used hydrologic routing method such as the Muskingum method.

## 9.1 The Muskingum Method

The Muskingum method of flood routing was developed in the 1930s in connection with the design of flood protection schemes in the Muskingum River Basin, Ohio, USA. It is the most widely used method of hydrologic river routing, with numerous applications throughout the world.

For a river channel reach where the water surface cannot be assumed horizontal, in case of flood flow, the stored volume becomes a function of the stages at both ends of the reach, and not at the downstream (outflow) end only.

In a typical reach, the different components of storage may be defined for a given instant in time as shown in Figure 9.1.

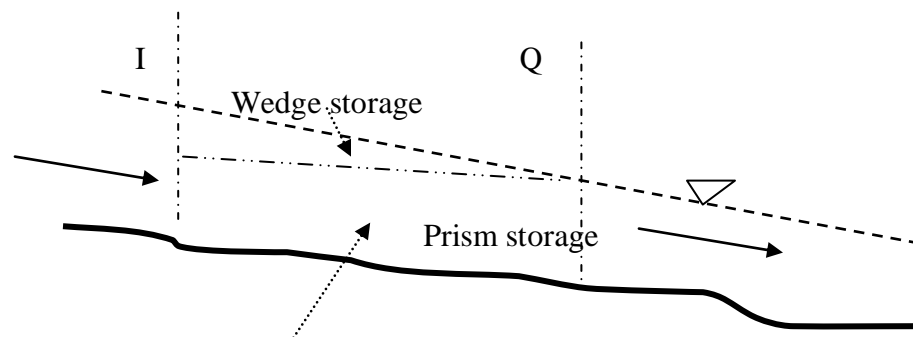


Fig. 9.1: River reach storages.

The continuity equation holds at any given time:  $dS/dt = I(t) - Q(t)$  where the total storage  $S$  is the sum of prism storage and wedge storage. The prism storage  $S_p$  is taken to be a direct function of the storage at the downstream end of the reach and the prism storage is the function of the outflow  $S_p = f_1(Q)$ . The wedge storage  $S_w$  exists because the inflow,  $I$ , differs from outflow  $Q$  and so may be assumed to be a function of the difference between inflow and outflow  $S_w = f_2(I - Q)$ .

The total storage may be represented by:

$$S = f_1(Q) + f_2(I - Q) \quad (9.1)$$

with due regard paid to the sign of the  $f_2$  term.

Assuming that in Eq. (9.1)  $f_1(Q)$  and  $f_2(I - Q)$  could be both a linear functions, i.e.  $f_1(Q) = KQ$  and  $f_2(I - Q) = b(I - Q)$ , we have

$$S = bI + (K-b)Q = K [(b/K)I + (1 - b/k)Q] \quad (9.2)$$

and writing  $X = b/K$ , we get

$$S = K [XI + (1 - X)Q] \quad (9.3)$$

$X$  is a dimensionless weighting factor indicating the relative importance of  $I$  and  $Q$  in determining the storage in the reach and also the length of the reach. The value of  $X$  has limits of zero and 0.5, with typical values in the range 0.2 to 0.4. Most streams have  $X$  values between 0.1 and 0.3.  $K$  has the dimension of time. Note that when  $X = 0$ , there is no wedge storage and hence no backwater, this is the case for a level-pool reservoir. The parameter  $K$  is the time of travel of the flood wave through the channel reach and should have the same time unit as  $\Delta t$ , the time interval of the inflow hydrograph.

The routing equation for the Muskingum method is derived by combining Eq. (9.1) and Eq. (9.3) and the result is given in Eq. (9.4). The condition is that  $C_1 + C_2 + C_3 = 1$ , and often the range for  $\Delta t$  is  $K/3 \leq \Delta t \leq K$ .

**Determination of  $K$ .** If observed inflow and outflow hydrograph are available for a river reach, the values of  $K$  and  $X$  can be determined. Assuming various values

$$Q_{j+1} = C_1 I_{j+1} + C_2 I_j + C_3 Q_j$$

where

$$C_1 = \frac{\Delta t - 2KX}{2K(1 - X) + \Delta t} \quad (9.4)$$

$$C_2 = \frac{\Delta t + 2KX}{2K(1 - X) + \Delta t}$$

$$C_3 = \frac{2K(1 - X) - \Delta t}{2K(1 - X) + \Delta t}$$

of  $X$  and using known values of the inflow and outflow, successive values of the numerator and denominator of the following expression can be computed:

$$K = \frac{0.5\Delta t[(I_{j+1} + I_j) - (Q_{j+1} + Q_j)]}{X(I_{j+1} - I_j) + (1 - X)(Q_{j+1} - Q_j)} \quad (9.5)$$

The computed values of the numerator and denominator are plotted for each time interval, with the numerator on the vertical axis and the denominator on the horizontal axis. This usually produces a graph in the form of a loop. The value of  $X$  that produces a loop closest to a single line is taken to be the correct value for the reach, and  $K$  is equal to the slope of the line. Finally  $K$  may be computed from the average value determined from Eq. (9.5) for the correct values of  $X$ . Note that since  $K$  is the time required for the incremental flood wave to traverse the reach, its value may also be estimated as the observed time of travel of peak flow through the reach.

With  $K = \Delta t$  and  $X = 0.5$ , flow conditions are such that the outflow hydrograph retains the same shape as the inflow hydrograph, but it is translated downstream a time equal to  $K$ . For  $X = 0$ , Muskingum routing reduces to linear reservoir routing.

As a guide  $K$  and  $X$  are chosen such that

$$X = \frac{0.5\Delta t}{K} \leq 1 - X \quad \text{and} \quad X \leq 0.5 \quad (9.6)$$

One rule of thumb used in practice is that the ratio  $\Delta t/K$  be approximately 1 and  $X$  be in the range 0 to 0.5 together with Eq. (9.6).

**Example 9.1:** Given the inflow and outflow hydrograph in Table E9.1 from Cols.1 to 3. Estimate the routing parameters  $K$  and  $X$ .

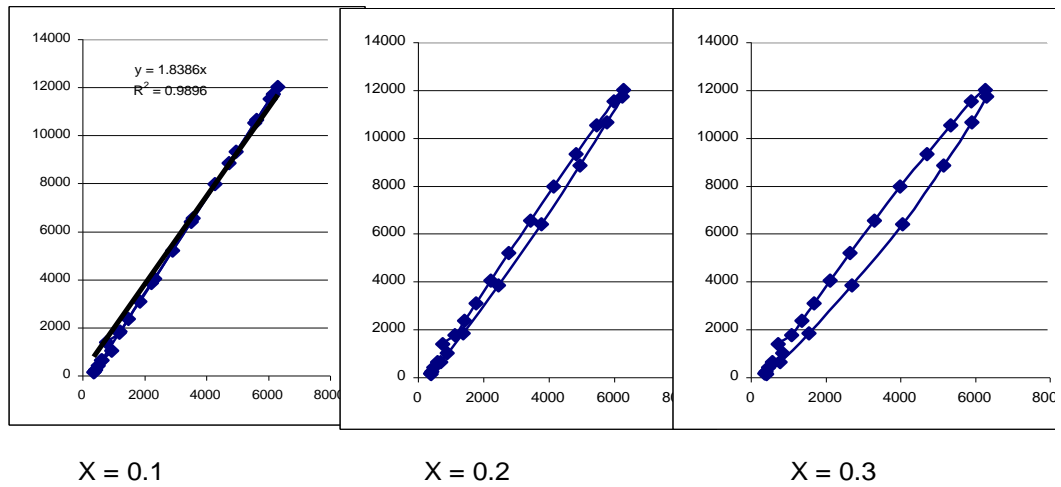
**Solution:** Channel storage is calculated by :

$$S_{j+1} = S_j + 0.5\Delta t[(I_{j+1} + I_j) - (Q_{j+1} + Q_j)]$$

Several values of  $X$  are tried, within the range 0.0 to 0.5, for example, 0.1, 0.2, and 0.3. For each trial value of  $X$ , the weighted flow  $(XI_j + (1-X)Q_j)$  are calculated, as shown in Table E9.1. Calculating the slope of the storage vs. weighted outflow curve then solves the value of  $K$ . In this case the value of  $K$  is 2 days for  $X = 0.1$ . It is to be noted that there is greater storage during the falling stage than during a rising stage of a flood for a given discharge.

Table E9.1. Derivation of  $X$  in the Muskingum method, example 9.1

Time (days)	Inflow (m3/s)	Outflow (m3/s)	Storage (m3/s).d	Weighted flow (m3/s)		
				X = 0.1	X=0.2	X = 0.3
0	352	352	0	0	0	0
1	587	383	102	403	424	444
2	1353	571	595	650	728	806
3	2725	1090	1803	1254	1417	1581
4	4408	2021	3814	2259	2498	2737
5	5987	3265	6369	3537	3809	4081
6	6704	4542	8812	4758	4974	5190
7	6951	5514	10611	5658	5801	5945
8	6839	6124	11687	6196	6267	6339
9	6207	6353	11972	6338	6323	6309
10	5346	6177	11483	6094	6011	5928
11	4560	5713	10491	5598	5482	5367
12	3861	5121	9285	4995	4869	4743
13	3007	4462	7928	4316	4171	4025
14	2358	3745	6507	3606	3467	3328
15	1779	3066	5170	2937	2809	2680
16	1405	2458	4000	2352	2247	2142
17	1123	1963	3054	1879	1795	1711
18	952	1576	2322	1513	1451	1389
19	730	1276	1737	1221	1167	1112
20	605	829	1352	807	784	762
21	514	1022	986	971	920	870
22	422	680	603	654	628	603
23	352	559	371	538	517	497
24	352	469	209	457	445	434
25	352	418	118	411	405	398



Where estimate of inflow and outflow hydrograph is not readily available standard practice is to assume a value of 0.2 for X, with a smaller value for

channel systems with large floodplains and larger values, near 0.4, for natural channels. The following relationships for estimating  $K$  and  $X$ :

$$K = \frac{0.6L}{V_o} \quad (9.10)$$

$$X = 0.5 - 0.3 \left( 1 - \frac{4F^2}{9} \right) \frac{y_o}{S_o L} \quad (9.6)$$

Where:

$L$  = the reach length (m)

$V_o$  = the average velocity (m/s)

$y_o$  = the full flow depth (m)

$S_o$  = the slope of the channel bottom (m/m)

$F$  = the Froude number  $V_o / [gy_o]^{1/2}$

**Example 9.2:** The flood hydrograph ordinates tabulated in Table E9.2 arrived at location A (Figure E9.2) on 20 July 1999 at 1:00 p.m. Determine the peak flow and arrival time (calendar day) of this flood at downstream location B. Muskingum coefficients of the reach from A to B are:  $X = 0.35$  and  $K = 1.2$  days. The initial outflow at B was  $10 \text{ m}^3/\text{s}$ .

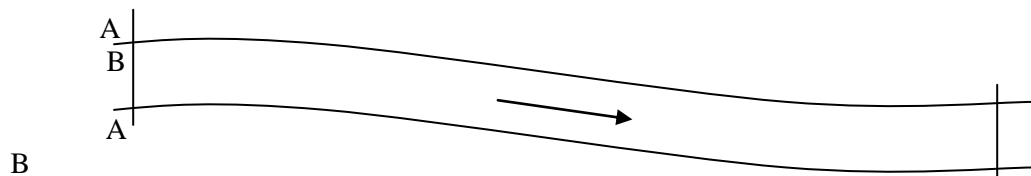


Figure E9.2. Schematics of a river reach (From A to B)

Table E9.2: Flood hydrograph ordinates at Section A-A.

Time (hr)	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
Inflow ( $\text{m}^3/\text{s}$ )	15	80	150	180	200	140	125	75	45	25

**Solution:**

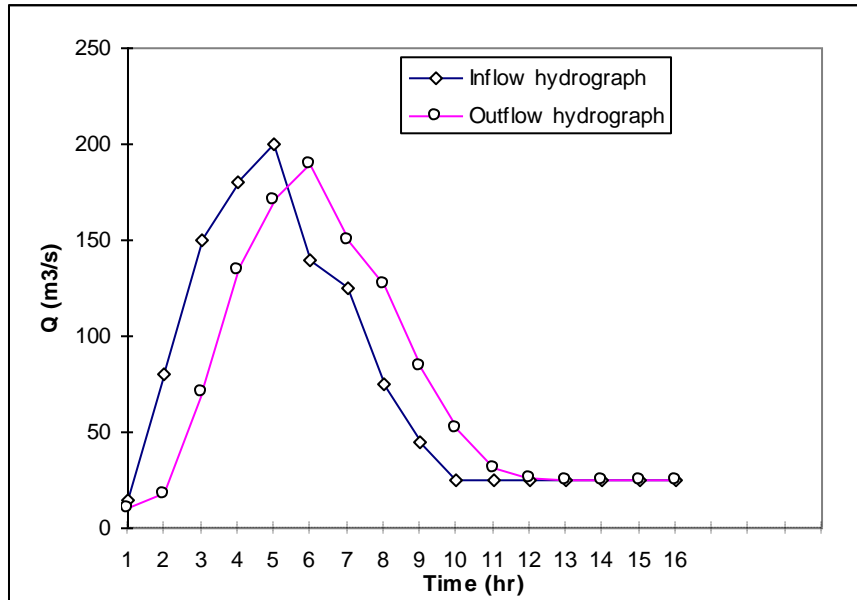
The Muskingum coefficients are determined for  $X = 0.35$  and  $K = 1.2$  hr. The values are  $C_1 = 0.0625$ ,  $C_2 = 0.7188$ , and  $C_3 = 0.2188$ . Then the outflow is calculated in Table E 9.2,

and plotted in Figure E9.2. The arrival time of the peak flood at section B-B is at 20 July 1999 at 6:00 p.m.

Table E9.2 Calculation of Outflow hydrograph from inflow hydrograph

1	2	3	4	5	6
Routing period j (hr)	Inflow I (m3/s)	C1 I <sub>j+1</sub>	C2 I <sub>j</sub>	C3 Q <sub>j</sub>	Outflow Q (m3/s)
Initial condition	10				10
1	15	0.94	7.19	2.19	10.31
2	80	5.00	10.78	2.26	18.04
3	150	9.38	57.50	3.95	70.82
4	180	11.25	107.81	15.49	134.55
5	<b>200</b>	12.50	129.38	29.43	171.31
6	140	8.75	143.75	37.47	<b>189.97</b>
7	125	7.81	100.63	41.56	149.99
8	75	4.69	89.84	32.81	127.34
9	45	2.81	53.91	27.86	84.57
10	25	1.56	32.34	18.50	52.41
11	25	1.56	17.97	11.46	31.00
12	25	1.56	17.97	6.78	26.31
13	25	1.56	17.97	5.76	25.29
14	25	1.56	17.97	5.53	25.06
15	25	1.56	17.97	5.48	25.01
16	25	1.56	17.97	5.47	25.00

Figure E9.2. Inflow and outflow hydrograph.





### 9.2 Practice problems

9.1. Given the following inflow and outflow hydrograph for a given reach, determine K and X.

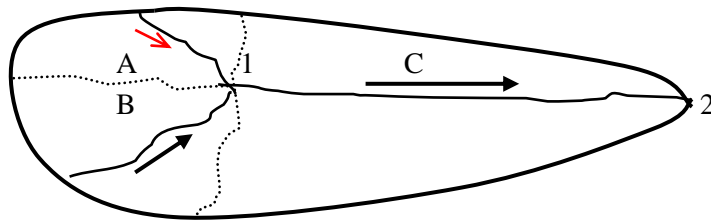
Time (hr)	1	2	3	4	5	6	7	8	9	10	11
Inflow (m <sup>3</sup> /s)	45	85	160	185	200	170	145	135	98	45	75
Outflow (m <sup>3</sup> /s)	45	50	75	88	145	160	175	145	78	56	40

9.2. Given the following inflow hydrograph to a certain stream channel reach, calculate the outflow hydrograph by the Muskingum method.

Time (hr)	1	2	3	4	5	6	7	8	9	10	11
Inflow (m <sup>3</sup> /s)	35	75	88	125	185	140	120	98	78	35	30

and K = 1 hr, X = 0.2 and Δt = 1 hr.

9.3. A storm event occurred on a given catchment that produced a rainfall pattern of 5 cm/hr for the first 10 min, 10 cm/hr in the second 10 min, and 5 cm/hr in the next 10 min. The catchment is divided into three sub-catchments.



The unit hydrograph of the three sub-catchments are given in the following table. Sub basins A and B had a loss rate of 2.5 cm/hr for the first 10 min and 1.0 cm/hr thereafter. Sub-basin C had a loss rate of 1.0 cm/hr for the first 10 min and 0 cm/hr thereafter. Determine the resulting flood at point 2 given the Muskingum coefficients K = 20 min and X = 0.2.

10 - minutes unit hydrographs

Sub-catchment A		Sub-catchment B		Sub-catchment C	
Time (min)	Q (m <sup>3</sup> /s/cm)	Time (min)	Q (m <sup>3</sup> /s/cm)	Time (min)	Q (m <sup>3</sup> /s/cm)
0	0	0	0	0	0
10	5	10	5	10	16
20	10	20	12	20	33
30	15	30	16	30	50
40	20	40	21	40	33
50	25	50	26	50	16
60	20	60	19	60	0
70	15	70	18		
80	10	80	10		
90	5	90	5		
100	0	100	0		

## 10. Reservoir Routing

Flood routing refers to the process of calculating the passage of a flood hydrograph through a system. It is a procedure to determine the time and magnitude of flow at a point on a downstream water course from known or assumed hydrograph at one or more points upstream. If the system is reservoir through which the flood is routed the term storage routing or reservoir routing is used.

Reservoir routing method is used:

- i. For flood forecasting in the lower parts of a river basin after passing through reservoir, the case of Awash river downstream of Koka dam,
- ii. For sizing spillways and determining dam / cofferdam height
- iii. For conducting river basin watershed studies for watersheds where one or more storage facilities exist. Specifically, for watersheds in which existing reservoir are located, a reservoir routing is necessary to evaluate watershed plans such as location of water supply structures, and regional flood control measures.

Note that in order to develop an operational flood routing procedures for a major river system, detailed knowledge of the main stream and the various feeder channels is necessary.

In comparison to other hydrological problems, storage routing is relatively complex. There are a number of variables involved, including (1) the input (upstream) hydrograph; (2) the output (downstream) hydrograph; (3) the stage-storage volume relationship measured from the site; (4) the energy loss (weir and orifice) coefficients; (5) physical characteristics (e.g., weir length, diameter of the riser pipe, length of the discharge pipe, etc.) of the outlet facility; (6) the storage volume versus time relationship; (7) the depth-discharge relationship; and (8) the target peak discharge allowed from the reservoir. The problem is further complicated in that the inflow and outflow hydrographs can be from either storms that have occurred (i.e., actually measured events) or design storm values.

## 10.1. Level pool or reservoir routing using storage indication or modified pulse method

Level pool routing is the procedure for calculating the outflow hydrograph from a reservoir with a horizontal water surface, given its inflow hydrograph and storage outflow characteristics.

For a hydrological system, input  $I(t)$ , output  $Q(t)$ , and storage  $S(t)$  are related by the continuity equation

$$\frac{dS}{dt} = I(t) - Q(t) \quad (10.1)$$

The time horizon is broken into intervals of duration  $\Delta t$ , indexed by  $j$ , that is  $t = 0, \Delta t, 2\Delta t, \dots, j \Delta t, (j+1)\Delta t, \dots$ , and the continuity equation is integrated over each time interval. For the  $j$ -th time interval:

$$\int_{S_j}^{S_{j+1}} dS = \int_{j\Delta t}^{(j+1)\Delta t} I(t)dt - \int_{j\Delta t}^{(j+1)\Delta t} Q(t)dt \quad (10.2)$$

The inflow values at the beginning and end of the  $j$ -th time interval are  $I_j$  and  $I_{j+1}$ , and the corresponding values of the outflow are  $Q_j$  and  $Q_{j+1}$ . If the variation of the inflow and outflow over the interval is approximately linear (for  $\Delta t$  small), the change in storage over the interval  $S_{j+1} - S_j$  can be found by rewriting the above

$$S_{j+1} - S_j = \frac{I_j + I_{j+1}}{2} \Delta t - \frac{Q_j + Q_{j+1}}{2} \Delta t \quad (10.3)$$

equation as

In order to solve the above equation let us separate group first the known (the right quantity from the equality) and the unknown (the left one) variables in the following equation:

$$\frac{2S_{j+1}}{\Delta t} + Q_{j+1} = (I_j + I_{j+1}) + \left( \frac{2S_j}{\Delta t} - Q_j \right) \quad (10.4)$$

The procedure then is first established storage-outflow relationship:  $2S/\Delta_t + Q$  and  $Q$  based on the existing storage-water elevation and outflow-water elevation data, physical characteristics of the reservoir.

The value of  $\Delta_t$  is taken as the time interval of the inflow hydrograph. For a given value of water surface elevation, the value of storage  $S$  and discharge  $Q$  are determined, then the value of  $2S/\Delta_t + Q$  is calculated and plotted.

In routing the inflow through time interval  $j$ , all terms on the right side of Eq. (10.4) are known, and so the values of  $2S_{j+1}/\Delta t + Q_{j+1}$  can be computed. The corresponding value of  $Q_{j+1}$  can be determined by linear interpolation of tabular values.

To set up the data required for the next time interval, the value of  $2S_{j+1}/\Delta t - Q_{j+1}$  is calculated by

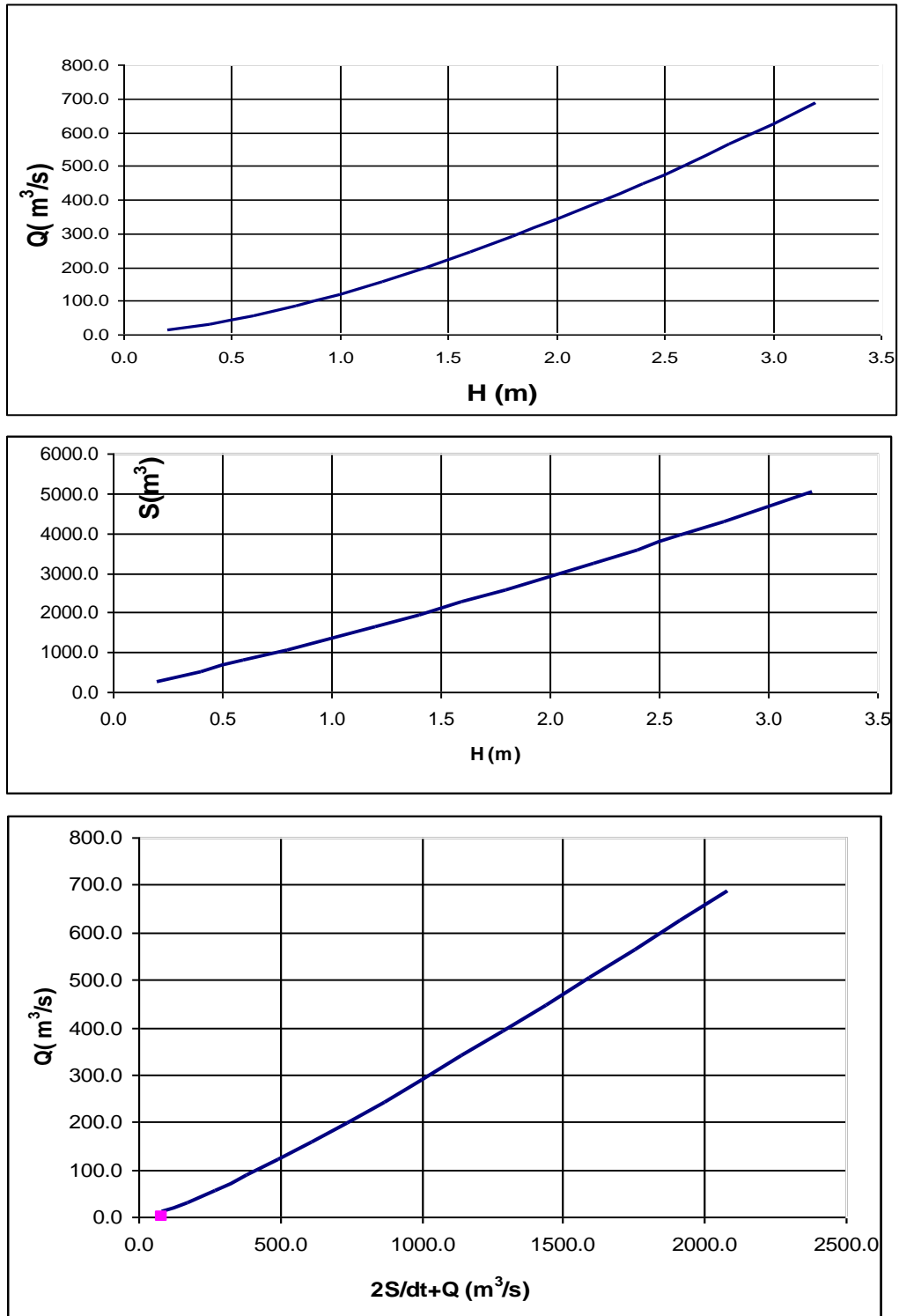
$$\left( \frac{2S_{j+1}}{\Delta t} - Q_{j+1} \right) = \left( \frac{2S_{j+1}}{\Delta t} + Q_{j+1} \right) - 2Q_{j+1} \quad (10.5)$$

The computation is then repeated for subsequent routing periods.

Depth (stage) storage relationships for a given contour lines can be computed as follows. The area within contour lines of the site can be planimetered, with the storage in any depth increment  $\Delta h$  equal to the product of the average area and the depth increment. Thus the storage increment  $\Delta S$  is given by:

$$\Delta S = 0.5 * (A_i + A_{i+1}) \Delta h \quad (10.6)$$

**Figure 10.1** Development of the storage-outflow function for level pool routing on the basis of storage-elevation and elevation-outflow curves.



**Example 10.1** . The design of an emergency spillway calls for a broad-crested weir of width  $L = 10.0$  m; rating coefficient  $C_d = 1.7$ ; and exponent  $n = 1.5$ . The spillway crest is at elevation 1070. Above this level, the reservoir walls can be considered to be vertical, with a surface area of 100 ha. The dam crest is at elevation 1076 m. Base flow is  $17 \text{ m}^3/\text{s}$ , and initially the reservoir level is at elevation 1071 m. Route the design hydrograph given in Table E10.2 through the reservoir. What is the maximum pool elevation reached?

**Solution.** The calculation of the storage indication function above the spillway crest elevation are shown in Table E10.1a. Outflow is calculated based on the  $Q = C_d L H^n = 1.7 * 10 * H^{1.5}$ .

The routing is summarized in Table E10.1b. The inflow hydrograph is given in Column 2 and 3. Columns 5 and 6 give calculated value of  $2S_j / \Delta t - Q_j$  and  $2S_{j+1} / \Delta t + Q_{j+1}$ . The initial outflow is  $17 \text{ m}^3/\text{s}$ ; the initial storage indication value (when the reservoir water level is 1 m above the spillway crest)

$$\frac{2S_{j+1}}{\Delta t} + Q_{j+1} = (I_j + I_{j+1}) + \left( \frac{2S_j}{\Delta t} - Q_j \right)$$

$$\text{for } j = 0 \quad \frac{2S_1}{\Delta t} + Q_1 = (I_0 + I_1) + \left( \frac{2S_0}{\Delta t} - Q_0 \right)$$

is  $= (17 + 17) + 2 * 100000 / 3600 - 17 = 572.56 \text{ m}^3/\text{s}$ , with corresponding outflow of  $Q_1 = 17 \text{ m}^3/\text{s}$ . For the next iteration for  $j = 1$ , we calculate  $\left( \frac{2S_1}{\Delta t} - Q_1 \right) = \left( \frac{2S_1}{\Delta t} + Q_1 \right) - 2Q_1$  value based on the last estimated value, that is  $2S_1 / \Delta t - Q_1 = 572.56 - 2 * 17 = 538.56 \text{ m}^3/\text{s}$ . Then

$$\frac{2S_{j+1}}{\Delta t} + Q_{j+1} = (I_j + I_{j+1}) + \left( \frac{2S_j}{\Delta t} - Q_j \right)$$

$$\text{for } j = 1 \text{ is } \frac{2S_2}{\Delta t} + Q_2 = (I_1 + I_2) + \left( \frac{2S_1}{\Delta t} - Q_1 \right)$$

$$= (17 + 20) + (538.56) = 575.56 \text{ m}^3/\text{s}.$$

The corresponding  $Q_2$  is then obtained from Table E10.1a as  $17.4 \text{ m}^3/\text{s}$ . The recursive procedure continues until iteration continues until the outflow has substantially reached the base flow condition. The maximum pool elevation (MPE) occurs at maximum spill of  $72.5 \text{ m}^3/\text{s}$ . It can be calculated from the ogee spillway equation and  $H = 2.63$  m depth, and the MPE is  $1070.0 + 2.63 = 1072.63$  m.

Table E10.1a Storage discharge relationship.

[1]	[2]	[3]	[4]	[5]	
Elevation (m)	Head above spillway crest (1070 m)	Q (outflow) m <sup>3</sup> /s	S	Storage (m <sup>3</sup> )	$2S_j+1/dt$ + $Q_{j+1}$ (m <sup>3</sup> /s)
1070.0	0.0	0.0		0	0
1071.0	1.0	17.0		1000000	573
1072.0	2.0	48.1		2000000	1159
1073.0	3.0	88.3		3000000	1755
1074.0	4.0	136.0		4000000	2358
1075.0	5.0	190.1		5000000	2968
1076.0	6.0	249.8		6000000	3583
1077.0	7.0	314.8		7000000	4204
1078.0	8.0	384.7		8000000	4829

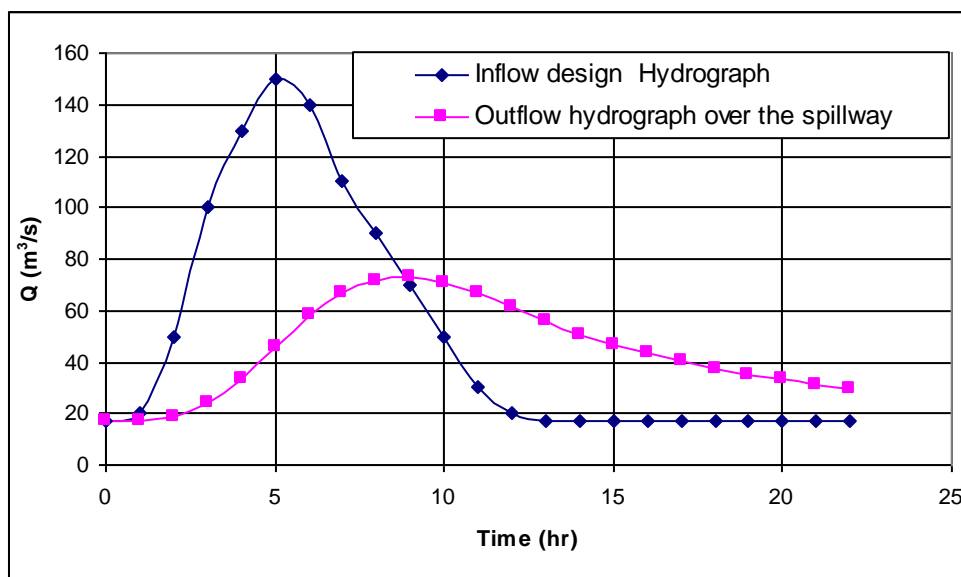


Table E10.1b: Design hydrograph and reservoir routing calculation

Time Index j	Time (hr)	Design Inflow hydrograph (m <sup>3</sup> /s)	I <sub>j</sub> +I <sub>j+1</sub> (m <sup>3</sup> /s)	2S <sub>j</sub> /dt - Q <sub>j</sub> (m <sup>3</sup> /s)	2S <sub>j+1</sub> /dt +Q <sub>j+1</sub> (m <sup>3</sup> /s)	Q <sub>j+1</sub> (m <sup>3</sup> /s)
1	0	17	34	538.6	572.6	17.0
2	1	20	37	538.6	575.6	17.4
3	2	50	70	540.7	610.7	19.0
4	3	100	150	572.7	722.7	24.3
5	4	130	230	674.1	904.1	33.7
6	5	150	280	836.7	1116.7	45.9
7	6	140	290	1024.8	1314.8	58.3
8	7	110	250	1198.2	1448.2	67.2
9	8	90	200	1313.9	1513.9	71.7
10	9	70	160	1370.5	1530.5	72.8
11	10	50	120	1384.9	1504.9	71.0
12	11	30	80	1362.8	1442.8	66.8
13	12	20	50	1309.2	1359.2	61.2
14	13	17	37	1236.8	1273.8	55.7
15	14	17	34	1162.4	1196.4	50.8
16	15	17	34	1094.8	1128.8	46.7
17	16	17	34	1035.5	1069.5	43.1
18	17	17	34	983.3	1017.3	40.1
19	18	17	34	937.2	971.2	37.4
20	19	17	34	896.3	930.3	35.2
21	20	17	34	860.0	894.0	33.2
22	21	17	34	827.6	861.6	31.4
23	22	17	34	798.8	832.8	29.9



### 10.1.2 Reservoir routing with controlled outflow

Most large reservoirs have some type of outflow control, wherein the amount of outflow is regulated by gated spillways. In this case, both hydraulic conditions and operational rules determine the prescribed outflow. Operational rules take into account the various use of water.

The differential equation of storage can be used to route flows through reservoirs with controlled outflow. In general, the outflow can be either (1) uncontrolled, (2) controlled (gated), or (3) a combination of controlled and uncontrolled. The discretized equation, including controlled outflow, is

$$\frac{S_{j+1} - S_j}{\Delta t} = \frac{I_j + I_{j+1}}{2} - \frac{Q_j + Q_{j+1}}{2} - Q_r \quad (10.6)$$

in which  $Q_r$  is the mean regulated outflow during the time interval  $\Delta t$ . With  $Q_r$  known, the solution proceeds in the same way as with the uncontrolled out flow case.

In the case where the entire outflow is controlled, Eq. (10.6) reduces to

$$S_{j+1} = S_j + \Delta t \frac{I_j + I_{j+1}}{2} - \Delta t Q_r \quad (10.7)$$

By which the storage volume can be updated based on average inflows and mean regulated outflow.

**Example 10.2** Discharge from a reservoir is over a spillway with discharge characteristic:

$$Q = 120 H^{1.5}$$

Where:  $Q$  in  $\text{m}^3/\text{s}$  and  $H$  is the head over the spillway (m).

The reservoir surface area is  $12.5 \text{ km}^2$  at spillway crest level and increases linearly by  $2 \text{ km}^2$  per meter rise of water level above crest level.

The design storm inflow, assumed to start with the reservoir just full, is given by a triangular hydrograph, base length 40 hr and a peak flow of 450 m<sup>3</sup>/s occurring after 18 hours after the start of flow. Estimate the peak outflow over the spillway and its time of occurrence to start of inflow (adapted from Shaw, 1994).

**Solution.** A level water surface in the reservoir is assumed. Temporary storage above the crest level is given by:

$$S = \int_0^H Adh = \int_0^H (12.5 + 2.0h)dh = 10^6 * (12.5H + H^2)(m^3)$$

Outflow over the crest are given by  $Q = 120H^{1.5}$

Taking the time interval of the inflow hydrograph 2 hr = 7200 s, we have

$$\frac{2S_{j+1}}{\Delta t} + Q_{j+1} = (I_j + I_{j+1}) + \left(\frac{2S_j}{\Delta t} - Q_j\right)$$

$$\frac{2S_{j+1}}{\Delta t} + Q_{j+1} = 2 * 10^6 * (12.5H_{j+1} + H_{j+1}^2) / 7200 + 120H_{j+1}^{1.5}$$

$$\frac{2S_{j+1}}{\Delta t} + Q_{j+1} = 34722H_{j+1} + 277.7H_{j+1}^2 + 120H_{j+1}^{1.5}$$

and

$$\frac{2S_{j+1}}{\Delta t} - Q_{j+1} = 34722H_{j+1} + 277.7H_{j+1}^2 - 120H_{j+1}^{1.5}$$

Now we need to derive the values of the above two functions.

## 10.2 Practice problems

10.1. Design the emergency spillway width (rectangular cross section) for the following dam, reservoir, and flood conditions: dam crest elevation = 483 m; emergency spillway crest elevation = 475 m, coefficient of spillway rating = 1.7; exponent of spillway rating = 1.5. Elevation-storage relation:

Elevation (m)	475	477	479	481	483
Storage (hm <sup>3</sup> )	5.1	5.3	5.6	6.4	7.6

Inflow hydrograph to reservoir:

Time (hr)	0	1	2	3	4	5	6	7	8	9	10	11	12	13
Inflow (m <sup>3</sup> /s)	0	10	30	50	60	150	250	350	280	210	190	170	130	100

Time (hr)	14	15	16	17	18	19	20	21	22	23	24
Inflow (m <sup>3</sup> /s)	90	75	50	40	30	15	10	5	2	1	0

Assume design freeboard = 3 m and initial reservoir pool level at spillway crest.

10.2 Solve Example 10.1 if the inflow hydrograph is changed to:

Time (min)	0	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150
Inflow (m <sup>3</sup> /s)	0.	2.7	4.4	6.9	8.7	9.4	15.1	19.6	20.9	10.7	8.6	5.5	4.4	2.1	1.5	0.