## Chapter two: Precipitation $\square$ What is precipitation?

* Precipitation is that part of atmospheric moisture that reaches the earth's surface in different forms
* The essential requirement of for precipitation to occur:
$\checkmark$ Moisture in the atmosphere
$\checkmark$ Presence of nuclei around which condensation vapor takes place
$\checkmark$ Dynamic cooling responsible for condensation of water vapor
$\checkmark$ Precipitation product must reach the ground in some form


## Cont

$\square$ Forms of precipitation (ppt):

* Rain: when ppt reaches the surface of the earth as droplets of water (size: 0.56mm)
* Snow: when ppt falls in the farm of ice-crystals (hexagonal in shape).
$\checkmark \quad$ It may fall separately or combines to form a flake
$\checkmark$ The density of snow is $0.1 \mathrm{~g} / \mathrm{cc}$
* Drizzle: water droplets of size less than 0.5 mm . It appears to be floating in the air
* Hail: it is the precipitating rain in the form of any irregular form of ice with size ranging from 5 to 50 mm
* Dew: during night when the surface of the objects on earth cools by radiation, the moisture present in the atmosphere condenses on the surface of these objects forming droplets of water


## Cont

$\square$ What are the types of precipitation?

* Precipitation formation is classified according to the factors responsible for lifting the air mass. The types of precipitation are:
$\checkmark$ Convective
$\checkmark$ Orographic
$\checkmark$ Cyclonic


## Cont.

$\checkmark$ Convective: lifting of unstable air that is warmer than surrounding air due to uneven surface heating
$>$ thunder storms
$>$ spotty and highly variable in intensity


## Cont

* Orographic: mountain range barriers cause lifting of the air masses
$\checkmark$ Moist air is forced over mountain barriers by westerly air flow \& ppt falls on wind (i.e., west) side of mountain range while the leeward (eastern) side is warmer and drier
> Medium to high intensity rainfall continuing for longer duration



## Cont.

* Cyclonic: a cyclone is a low pressure area surrounded by a larger high pressure area.
$\checkmark$ When a low pressure occurs in an area, especially over large water bodies It causes the air at the low pressure zone to lift
$>$ The name of this cyclones vary in different places
- Tropical cyclone (in India)
- Typhoon (South East Asia)
- Hurricane (America)
cloud


Moist air flows from high pressure area to low pressure area

## Cont.....

$\square$ Measurement of precipitation

* Precipitation is measured as a vertical depth of water that would accumulate on the horizontal surface
* Units of measurement: mm, cms, inches, feet, etc



## Cont....

* It is collected \& measured by in instrument called rain gauge
* Instruments that measure precipitation are:


## $>$ Non-recording

- cylindrical container type



## Cont....

Rainfall Recorders
$>$ Self recording

- weighing type
- float type



## Cont......

$>$ Remotely sensed techniques
> Satellites are in the unique position of being able to provide a broad, spatially consistent, and continuous global sampling of many essential climate variables like precipitation


## Cont....

The Global Satellite Observation System


## Cont...

- Radar



## Cont.....

## Weather Radar

- Measurement of rainfall by weather radar based on principle of that the amount of power returned from raindrop is related to rainfall intensity. The advantages are:
- Total aerial rainfall can be computed in near real time using online computers at the weather station
- Can be used for flood forecasting
- The movement of weather system can be tracked and shown
- can forecast time of onset, cession and rainfall intensity
- Dangerous meteorological phenomena can be detected


## Cont...

## - Example

Satellite rainfall estimation of Tigray Regon in the first ten days of June 2003


## Cont....

- Example

Satellite rainfall estimation of Tigray Regon in the second ten days of July 2003


Actual Rainfall in mm
$>0.0$ to 10.0
$>10.0$ to 20.0
$>20.0$ to 30.0
$>30.0$ to 50.0
$>50.0$ to 100.0
$>100.0$ to 200.0
$>200.0$ to 300.0

## Cont...

$\square$ Ideal Location for a Rain Gauge Station
$>$ While setting up any rain Gauge station the following points should be noted.
I. The site should be on a level ground, i.e., slopping ground, hill tops or hill slopes are not suitable.
II. The site should be an open space.
III. Horizontal distance between the rain gauge and the nearest objects should be twice the height of the objects.
IV. Site should be away from continuous wind forces.
V. Other meteorological instruments and the fencing of the site should maintain the step III) above.
VI. The site should be easily accessible.
VII. The gauge should be truly vertical.
VIII. Ten percent of total number of rain gauge stations of any basin should be self-recording.
IX. The observer must visit the site regularly to ensure its proper readiness for measurement.

## Cont..

- Location of meteorological stations



## Cont....

$\square$ Precipitation data errors

* The most significant cause of error in rain data
$>$ Observations are usually turbulent airflow around the gauge.
$>$ The dependence of rain gauge catch on wind speed
> Evaporation from within the measuring cylinder
$>$ Adhesion of water to funnel, (water then evaporates)
> Gauge leaks, blockages or overflows in storms
- Error in observation or transcription
$>$ Splash in or out


## Cont...

$\square$ Estimating missing data and of Adjustment of records

* Complete measured precipitation data are important to many problems in hydrologic analysis and design but there are missing values
* The causes of missing rainfall data are:
$>$ The failure of the observer to make the necessary visit to the gage may result in missing data.
$>$ Vandalism of recording gages is another problem that results in incomplete data records
$>$ Instrument failure because of mechanical or electrical malfunctioning can result in missing data.


## Cont...

Some of the methods of estimating missing rainfall values are

1. Station-Average Method
2. Normal-Ratio Method
3. Quadrant Method
4. Regression method

## Cont...

* Station-Average Method
- The station-average method for estimating missing data uses n gages from a region to estimate the missing point rainfall, P , at another gage

Where

$$
\hat{P}=\frac{1}{n} \sum_{i=1}^{n} P_{i}
$$

- Pi is the catch at gage $i$.
- Equation is conceptually simple, but may not be accurate when the total annual catch at any of the n regional gages differ from the annual catch at the point of interest by more than $10 \%$.

Cont.....

## Example, consider the following data

| Gage | Annual <br> precipitation(mm) | Monthly precipitation <br> $(\mathrm{mm})$ |
| :--- | :--- | :--- |
| A | 420 | 26 |
| B | 410 | 31 |
| C | 390 | 23 |
| X | 410 | $?$ |

## Cont..

- Ten percent of the annual catch at gage $X$ is 41 mm and the average annual catch at each of the three regional gages is within + or - 41 mm .; therefore, the station-average method can be used. The estimated catch at the gage with the missing monthly precipitation total is


## $\tilde{P}=1 / 3(26+31+23)=26.667 \mathrm{~mm}$

- This method is often used in flat areas with very less rainfall variability

Cont.

* Normal-Ratio Method

Where

$$
\hat{P}=\sum_{i=1}^{n} w_{i} P_{i}
$$

- $w i=$ the weight for the rainfall depth Pi at gage i . The weight for station i is computed by

$$
w_{i}=\frac{A_{x}}{n A_{i}}
$$

Where

- $\mathrm{Ai}=$ the average annual catch at gage i ,
- $\mathrm{Ax},=$ the average annual catch at station X ,
- $\mathrm{n}=$ the number of stations.
- Note:

When the average annual catches differ by more than $10 \%$, the normal-ratio method is preferable; such differences might occur in regions where there are large differences in elevation (for example, regions where orographic effects are present) or where average annual rainfall is low but has high annual variability.

Cont...

- Example

To illustrate the normal-ratio method, consider the following data:

| Gage | Annual <br> precipitation(mm) | Monthly precipitation <br> $(\mathrm{mm})$ |
| :--- | :--- | :--- |
| A | 410 | 24 |
| B | 370 | 23 |
| C | 460 | 31 |
| X | 400 | $?$ |

Determine the monthly precipitation at gage X

Cont.....

- The monthly precipitation for gage X is missing and can be estimated using the data from the table
- The steps are
I. Calculate ten percent of annual precipitation at gage X which is 40
II. Add and subtract 40 from the annual precipitation of gage $X$ to determine the range which is from 360 mm to 440 mm
III. Check whether all the annual precipitation of the stations are with in the above range
IV. If at least one station is outside the rage, use normal ratio method. As indicated in the table the annual precipitation of gage C is 460 mm which is outside the range there fore we use normal ratio method to determine the missing data at station X

Cont.

- This how the missing value at gage X is calculated

$$
\begin{gathered}
\hat{P}=w_{A} P_{A}+w_{B} P_{B}+w_{C} P_{C} \\
=\frac{A_{x}}{n A_{A}} P_{A}+\frac{A_{x}}{n A_{B}} P_{B}+\frac{A_{x}}{n A_{C}} P_{C} \\
=\quad \mathrm{Ax} / \mathrm{n}(\mathrm{PA} / \mathrm{AA}+\mathrm{PB} / \mathrm{AB}+\mathrm{PC} / \mathrm{AC}) \\
=\quad 400 / 3(24 / 410+23 / 370+31 / 460) \\
=25.08
\end{gathered}
$$

Cont...

- Inverse distance method(U.S. weather Service method)

The missing rainfall data of the station $X$ is computed by the following equation.

$$
p_{x}=\frac{\sum_{i=1}^{n} p_{i} * w_{i}}{\sum_{i=1}^{n} w_{i}}
$$

where
$W_{i}=\frac{1}{D_{i}{ }^{2}}=\frac{1}{X_{i}{ }^{2}+Y_{i}{ }^{2}}$
$p_{i}=$ the rainfall of the surrounding stations
$\left(x_{i}, y_{i}\right)=$ the coordinate of each surrounding station $(0,0)=$ the coordinate of the station with missing data
Note
This method considers the distance of the stations.

- Example

In a river basin a station A was in operative during a storm, while stations B,C,D and E , surrounding A were in operation, recording $74 \mathrm{~mm}, 88 \mathrm{~mm}, 71 \mathrm{~mm}$ and 80 mm of monthly rainfall. The coordinates of the stations is given in the figure below. Estimate the missing monthly rainfall of station A by the inverse distance method.


| Stations |  | Yi | xi2 | $\mathrm{yi}^{2}$ | $\mathrm{xi}^{2}+\mathrm{yi}^{2}=\mathrm{Di}$ | wi=1/Di | Pi(mm) | wi*Pi |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| B | 9 | 6 | 81 | 36 | 117 | 0.008547 | 74 | 0.632479 |
| C | 12 | -9 | 144 | 81 | 225 | 0.004444 | 88 | 0.391111 |
| D | -11 | -6 | 121 | 36 | 157 | 0.006369 | 71 | 0.452229 |
| E | -7 | 7 | 49 | 49 | 98 | 0.010204 | 80 | 0.816327 |
| sum |  |  |  |  |  | 0.029565 |  | 2.292146 |
| Px | 77.5291 |  |  |  |  |  |  |  |

REGRESSION method for filling missing data

- Regression analysis is a procedure for fitting an equation to a set of data. Specifically, given a set of measurements on two random variables, y and x , regression provides a means for finding the values of the coefficients $a$ and $b$ for the straight line $(y=a+b x)$ that best fits the data.
The coefficients $a$ and $b$ can be found using least square method using the following two equations simultaneously

$$
\begin{gathered}
\mathrm{na}+\mathrm{b} \sum x=\sum y \\
\mathrm{a} \sum x+\mathrm{b} \sum x^{2}=\sum x y
\end{gathered}
$$

- Example

The rainfall (RF) data of the table below are the annual rainfall (mm) and monthly rainfall ( mm ) of a meteorological station from 1990-1995 E.C. Estimate the monthly rainfall value of the station for the year 1996 E.C. using the regression method.

| Data of a station |  |  |
| ---: | ---: | ---: |
| Year | Annual RF(mm) | monthly RF(mm) |
| 1990 | 1100 | 200 |
| 1991 | 1200 | 220 |
| 1992 | 1000 | 180 |
| 1993 | 950 | 170 |
| 1994 | 1150 | 210 |
| 1995 | 1250 | 230 |
| 1996 | 1300 | $x$ |

- Solution
$>$ Develop an equation in the form of $y=a+b x$, That is

$$
\text { monthly } \mathrm{RF}=\mathrm{a}+\mathrm{b}^{*} \text { annual } \mathrm{RF}
$$

$>$ The coefficients a and b can be found using the equations

$$
\begin{gathered}
\mathrm{na}+\mathrm{b} \sum x=\sum y \\
\mathrm{a} \sum x+\mathrm{b} \sum x^{2}=\sum x y
\end{gathered}
$$

| Data of a station |  |  |  |  |
| :---: | ---: | ---: | ---: | ---: |
| Year | Annual RF(mm)(X) | monthly RF(mm)(y) | x2 | xy |
| 1990 | 1100 | 200 | 121000 | 220000 |
| 1991 | 1200 | 220 | 1440000 | 264000 |
| 1992 | 1000 | 180 | 1000000 | 180000 |
| 1993 | 950 | 170 | 902500 | 161500 |
| 1994 | 1150 | 210 | 1322500 | 241500 |
| 1995 | 1250 | 230 | 1562500 | 287500 |
| 1996 | 1300 | x |  |  |
| Sum | 6650 | 1210 | 7437500 | 1354500 |

## Cont..

- There fore, the equations become

$$
\begin{gather*}
6 * a+b * 6650=1210-----------------------------(1) \\
a * 6650+b * 7437500=1354500-----(2) \tag{2}
\end{gather*}
$$

Solving the two equations simultaneously
$\mathrm{a}=-20$ and $\mathrm{b}=0.2$ in which $\mathrm{y}=-20+0.2 \mathrm{x}$ or monthly rainfall $=-20+0.2 *$ annual rainfall
To estimate the monthly rainfall for the year 1996 , substitute the annual rainfall of the year in to the equation and is estimated as monthly rainfall $(\mathrm{mm})=-20+0.2 * 1300 \mathrm{~mm}$

$$
=240 \mathrm{~mm}
$$



- Adjustment of rainfall records (gage consistency)
- A consistent record is one where the characteristics of the record have not changed with time
- An inconsistent record may result from any one of a number of events
- Lack of consistence may be due to
- Unreported shifting the rain gauge (by as much as 8 km aerially \& 3 m in elevation)
- Significant construction work might have changed the surrounding
- change of observational procedure
* Double-mass-curve analysis is the method that is used to check for an inconsistency in a gaged record.
* A double-mass curve is a graph of the cumulative catch at the rain gage of interest versus the cumulative catch of one or more gages in the regions that have been subjected to similar hydro meteorological occurrences and are known to be consistent
* If a double- mass curve has a constant slope, the record is consistent.
* If a double -mass curve has not a constant slope, the record is not consistent and need to be adjusted.
- Steps to check and adjust the consistency of rainfall data of a station X
I. The doubtful station, say $A$, is marked and the group of stations surrounding it are identified.
II. Determine the cumulative rainfall of the station A
III. Sum the data of the neighboring stations and determine the cumulative
IV. Plot the sum cumulative rainfall of stations with cumulative station A to determine the double mass curve
V. If there is a changing slope in the double-mass curve, adjust it correct the data of station A by the adjustment factor
- Say the slopes of the two sections, S1, and S2, can be computed from the cumulative catches:

- Adjustment of the initial data of a station
- $\frac{S 2}{S 1}$ is the adjustment factor and multiplying each value in the Y1series by the adjustment factor


$$
y_{1}=\frac{S_{2}}{S_{1}} Y_{1}
$$

(b) Adjustment of lower section

- Example: Adjustment of the Lower Section of a Double -Mass Curve.
- In the table below annual rainfalls of gage $\mathrm{E}, \mathrm{F}, \mathrm{G}$ and H are given. Gage H was permanently relocated after a period of 3 yr (at the end of 1981); thus adjust the recorded values from 1979 through 1981 of gage H using double mass curve assume the data for the other gages are consistent.

|  | Annual Catch <br> (in.) at Gage |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
| Year | $E$ | $F$ | $G$ | $H$ |
| 1979 | 22 | 26 | 23 | 28 |
| 1980 | 21 | 26 | 25 | 33 |
| 1981 | 27 | 31 | 28 | 38 |
| 1982 | 25 | 29 | 29 | 31 |
| 1983 | 19 | 22 | 23 | 24 |
| 1984 | 24 | 25 | 26 | 28 |
| 1985 | 17 | 19 | 20 | 22 |
| 1986 | 21 | 22 | 23 | 26 |

- Solution

|  | Annual Catch <br> (in.) at Gage |  |  |  |  |  | Cumulative Catch <br> (in.) for Gage |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  | $F$ | $G$ | $H$ |  | TOTAL | $E+F+G$ | $E+F+G$ |

- Plot

The slope for the 1979-1981 period is

$$
S_{1}=\frac{99-0}{229-0}=0.4323
$$

The slope from 1982 to 1986 is

$$
S_{2}=\frac{230-99}{573-229}=0.3808
$$

the adjusted values from 1979 through 1981 can be computed using

$$
h_{1}=\left(\frac{0.3808}{0.4323}\right) H_{1}=0.8809 H_{1}
$$

- Example Adjustment of the Upper Section of a Double-Mass Curve
- Data for gages $A, B$, and $C$ and $D$ are given the table below. Check the consistence of data for gage D and adjust it using double-mass curve method.

|  | Annual Catch <br> (in.) at Gage |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
| Year | $A$ | $B$ | $C$ | $D$ |
| 1977 | 30 | 31 | 27 | 29 |
| 1978 | 33 | 36 | 32 | 32 |
| 1979 | 26 | 27 | 24 | 28 |
| 1980 | 27 | 26 | 27 | 29 |
| 1981 | 34 | 34 | 30 | 30 |
| 1982 | 31 | 33 | 31 | 29 |
| 1983 | 28 | 30 | 24 | 28 |
| 1984 | 35 | 34 | 33 | 39 |
| 1985 | 37 | 39 | 36 | 41 |
| 1986 | 34 | 35 | 35 | 37 |

Cont..

- Soultion

| Year | Annual Catch (in.) at Gage |  |  |  | $\begin{aligned} & \text { TOTAL } \\ & A+B+C \end{aligned}$ | Cumulative Catch (in.) |  | $d_{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | A | B | C | D |  | $A+B+C$ | D |  |
| 1977 | 30 | 31 | 27 | 29 | 88 | 88 | 29 |  |
| 1978 | 33 | 36 | 32 | 32 | 101 | 189 | 61 |  |
| 1979 | 26 | 27 | 24 | 28 | 77 | 266 | 89 |  |
| 1980 | 27 | 26 | 27 | 29 | 80 | 346 | 118 |  |
| 1981 | 34 | 34 | 30 | 30 | 98 | 444 | 148 |  |
| 1982 | 31 | 33 | 31 | 29 | 95 | 539 | 177 |  |
| 1983 | 28 | 30 | 24 | 28 | 82 | 621 | 205 |  |
| 1984 | 35 | 34 | 33 | 39 | 102 | 723 | 244 | 35.0 |
| 1985 | 37 | 39 | 36 | 41 | 112 | 835 | 285 | 36.8 |
| 1986 | 34 | 35 | 35 | 37 | 104 | 939 | 322 | 33.2 |

Cont...

- Plot

the slope for the 1977-1983 period is

$$
S_{1}=\frac{205-0}{621-0}=0.3301
$$

The slope for the 1984-1986 period is

$$
S_{2}=\frac{322-205}{939-621}=0.3679
$$

the equation for adjusting the upper section of the curve is

$$
d_{2}=\left(\frac{0.3301}{0.3679}\right) D_{2}=0.8973 D_{2}
$$

$\square$ Mean Areal Depth of Rainfall (Average Areal Rainfall)

* Two of the more common problems where this type of analysis could be used are
$\checkmark \quad$ In the computation of the probable maximum flood for larger watersheds and
$\checkmark$ in the analysis of actual storm events in which the rainfall depths were measured at more than one rain gage.
* Average Areal Rainfall over an area is found using the existing stations using interpolation techniques ( extending the point rainfall to areal estimates) and helps to estimate the rainfall amount at the ungaged areas.
* Some of the methods for estimating the areal rainfall of an area from point rainfalls are
$>$ Station-Average Method
> Theissen Polygon Method
> Isohyetal Method

I. Station-Average Method (Arithmetic mean Method)

$$
\bar{P}=\frac{1}{n} \sum_{i=1}^{n} P_{i}
$$

```
Where
    P}=\mathrm{ the average rainfall,
    Pi}=\mathrm{ the rainfall measured at station i
    n= the number of rain gages, and
```

Note
The use of this Equation will provide reasonably accurate estimates of when there are no Significant orographic effects, the gages are uniformly spaced throughout the watershed, and the rainfall depth over the entire watershed is nearly constant.

Cont....

## Theissen Polygon Method (nearest point method)



* The steps for thiessen polygon method are:
- Construct polygons by connecting stations with lines
- Bisect the polygon sides
- Estimate the area of each stations polygon
- Sum the areas
- Determine the stations weights by dividing the station area by the total area
- Determine areal precipitation by summing weighted precipitation for each station
* Theissen polygone method
- Unique for each gage network
- Allows for areal weighing of precipitation data
- Does not allow for orographic effects (those due to elevation changes)
- Most widely used method for cases where there are large differences in the catches at the rain gages and/or the rain gages are not uniformly distributed throughout the watershed,

Cont...

## Examples

The area shown in Fig. below is composed of a square plus an equilateral triangular plot of side 10 km . The annual precipitations at the rain-gauge stations located at the four corners and center of the square plot and apex of the triangular plot are indicated in the figure. Find the mean precipitation over the area by Thiessen polygon method, and compare with the station average method (arithmetic mean method)


Cont.....

* Solution
$\checkmark$ Using Thiessen polygon method
The Thiessen polygon is constructed by drawing perpendicular bisectors to the lines joining the rain-gauge stations as shown in Fig. below. The weighted mean precipitation is computed in the following table


Cont...

- The area of each polygon are

Area of square plot $=10 \times 10=100 \mathrm{~km} 2$
Area of inner square plot $=\frac{10}{\sqrt{2}} * \frac{10}{\sqrt{2}}=50 \mathrm{~km} 2$
Difference $=50 \mathrm{~km} 2$
Area of each corner triangle in the square plot $=\frac{56}{4}=12.5 \mathrm{~km} 2$
$\frac{1}{3}$ area of the equilateral triangular plot $=\frac{1}{3}\left(\frac{1}{2} * 10^{*} 10^{*} \sin 60\right)=14.4 \mathrm{~km} 2$

Cont....

| station | Area, A(km^2) | Precipitation(cm) | A*P(km^2-cm) | Pave |
| :---: | :---: | :---: | :---: | :---: |
| A | $(12.5+14.4)=26.9$ | 46 | 1238 | $\frac{\sum A * P}{\sum A}=66.3 \mathrm{~cm}$ |
| B | 12.5 | 65 | 813 |  |
| C | 12.5 | 76 | 950 |  |
| D | $(12.5+14.4)=26.9$ | 80 | 2152 |  |
| E | 50 | 70 | 3500 |  |
| F | 14.4 | 60 | 864 |  |
| $\mathrm{n}=6$ | $\sum A=143.2$ | $\sum P=397$ | $\sum A * P=9517$ |  |

- Using arithmetic mean method

$$
\bar{P}=\frac{1}{n} \sum_{i=1}^{n} P_{i}
$$

Pave $=\frac{397}{6}=66.17 \mathrm{~cm}$ which compares fairly with the weighted mean.
$\square$ Isohyetal Method

$$
\bar{P}=\sum_{i=1}^{n}\left(\frac{A_{i}}{A}\right) \widehat{P_{i}}
$$

Where
$\mathrm{Ai}=$ the area of the watershed between isohyets i and $\mathrm{i}+1$, $\mathrm{Pi}=$ the average precipitation for isohyets i and $\mathrm{i}+1$, and $\mathrm{n}=$ the number of isohyetal intervals.
$\frac{A i}{A}=$ the weight applied to the particular precipitation range.

Cont.....

- Steps for the isohyetal method
I. Draw lines of equal precipitation or rainfall (isohyets)
II. Estimate the precipitation between the isohyets
III. Determine the area between the isohyets
IV. Multiply step II and III and sum the values
V. Divide the sum by the total area of the basin to get area rainfall estimation

Note
The isohyetal method considers topographical effects but it is difficult to draw the isohyets

## Example

- Point rainfalls due to a storm at several rain-gauge stations in a basin are shown in Fig. blow. Determine the mean areal depth of rainfall over the basin by the isohyetal method.


Cont...

- Solution
- The isohyets are drawn as shown below and the mean areal depth of rainfall is worked out in the table below


Cont...

- Here is the worked out

| Zone | Isohyets (cm) | Mean isohyetal value, $P_{l-2}$ (cm) | Area between isohyets, $A_{l-2}$ $\left(\mathrm{km}^{2}\right)$ | Product (3) $\times$ (4) $\left(\mathrm{km}^{2} \cdot \mathrm{~cm}\right)$ | Mean areal depth of rainfall (cm) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 2 | 3 | 4 | 5 | 6 |
| I | <6 | 5.4 | 410 | 2214 | $P_{\text {ave }}=$ |
| II | 6-8 | 7 | 900 | 6300 | $\Sigma A_{1-2} P_{1-2}$ |
| III | 8-10 | 9 | 2850 | 25650 | $\Sigma A_{1-2}$ |
| IV | 10-12 | 11 | 1750 | 19250 | 66754 |
| V | >12 | 12.8 | 720 | 9220 | 7180 |
| VI | <8 | 7.5 | 550 | 4120 | $=930 \mathrm{~cm}$ |
|  |  | Total | $\begin{gathered} 7180 \mathrm{~km}^{2} \\ =\Sigma A_{1-2} \end{gathered}$ | $\begin{gathered} 66754 \mathrm{~km}^{2} \cdot \mathrm{~cm} \\ =\Sigma A_{1-2} P_{1-2} \end{gathered}$ |  |

## OPTIMUM RAIN-GAUGE NETWORK DESIGN

- When the mean areal depth of rainfall is calculated by the simple arithmetic average, the optimum number of rain-gauge stations to be established in a given basin is given by the equation.

$$
N=\left(\frac{C_{v}}{p}\right)^{2}
$$

Where
$\mathrm{N}=$ optimum number of rain gauge stations to be established in the basin
$\mathrm{Cv}=$ Coefficient of variation of the rainfall of the existing rain gauge stations (say, n)
$p=$ desired degree of percentage error in the estimate of the average depth of rainfall over the basin
The number of additional rain-gauge stations ( $\mathrm{N}-\mathrm{n}$ ) should be distributed in the different zones (caused by isohyets) in proportion to their areas, i.e., depending upon the spatial distribution of the existing rain-gauge stations and the variability of the rainfall over the basin

## Example

For the basin shown in Fig below, the normal annual rainfall depths recorded and the isohyetals are given. Determine the optimum number of rain-gauge stations to be established in the basin if it is desired to limit the error in the mean value of rainfall to $10 \%$. Indicate how you are going to distribute the additional rain-gauge stations required, if any. What is the percentage accuracy of the existing network in the estimation of the average depth of rainfall over the basin?


Cont...

- solution

| Station | Normal annual rainfall, $x$ ( cm ) | Difference $(x-\bar{x})$ | $\begin{aligned} & \text { (Difference) }^{2} \\ & (x-\bar{x})^{2} \end{aligned}$ | Statistical parameters $\bar{x}, \sigma, C_{v}$ |
| :---: | :---: | :---: | :---: | :---: |
| A | 88 | -4.8 | 23.0 | $\bar{x}=\frac{\Sigma x}{n}=\frac{464}{5}$ |
| B | 104 | 11.2 | 125.4 | $=92.8 \mathrm{~cm}$ |
| C | 138 | 45.2 | 2040.0 | $\sigma=\sqrt{\frac{\Sigma(x-\bar{x})^{2}}{n-1}}$ |
| D | 78 | -14.8 | 219.0 | $=\sqrt{\frac{3767.4}{5-1}}=30.7$ |
| E | 56 | -36.8 | 1360.0 |  |
| $n=5$ | $\Sigma x=464$ |  | $)^{2}=3767.4$ | $\begin{aligned} C_{v}=\frac{\sigma}{\bar{x}} & =\frac{30.7}{928} \times 100 \\ & =33.1 \% \end{aligned}$ |

Cont....
The optimum number of rain-gauge stations to limit the error in the mean value of rainfall to $\mathrm{p}=$ 10\%

$$
N=\left(\frac{C_{1}}{p}\right)^{2}=\left(\frac{33.1}{10}\right)^{2}=11
$$

$\therefore$ Additional rain-gauge stations to be established $=\mathrm{N}-\mathrm{n}=11-5=6$ The additional six rain gauge stations have to be distributed in proportion to the areas between the ishhyetals as shown below

Cont...

| Zone | $I$ | $I I$ | III | $I V$ | $V$ | $V I$ | Total |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Area $\left(\mathrm{Km}^{2}\right)$ | 410 | 900 | 2850 | 1750 | 720 | 550 | 7180 |
| Area, as decimal | 0.06 | 0.12 | 0.40 | 0.24 | 0.10 | 0.08 | 1.00 |
| $\mathrm{~N} \times$ Area in |  |  |  |  |  |  |  |
| decimal $(\mathrm{N}=11)$ | 0.66 | 1.32 | 4.4 | 2.64 | 1.1 | 0.88 |  |
| Rounded as | 1 | 1 | 4 | 3 | 1 | 1 | 11 |
| Rain-gauges existing | 1 | 1 | 1 | 1 | 1 | - | 5 |
| Additional rain <br> gauges | - | - | 3 | 2 | - | 1 | 6 |

These additional rain-gauges have to be spatially distributed between the different isohyetals after considering the relative distances between rain-gauge stations, their accessibility, personnel required for making observations, discharge sites, etc.

Cont....
The percentage error p in the estimation of average depth of rainfall in the existing network,

$$
\begin{aligned}
& p=\frac{C_{v}}{\sqrt{N}}, \text { putting } N=n \\
& p=\frac{33.1}{\sqrt{5}}=14.8 \%
\end{aligned}
$$

Or, the percenilage accuracy $=85.2 \%$

## ANALYSIS OF RAINFALL DATA

- Rainfall during a year or season (or a number of years) consists of several storms. The characteristics of a rainstorm are:
I. Intensity (cm/hr)
II. Duration (min, hr, or days)
III. Frequency (once in 5 years or once in 10, 20, 40, 60 or 100 years), and
IV. Areal extent (i.e., area over which it is distributed).


## Intensity, Duration and Frequency (IDF) curves

* An IDF is a three parameter curve, in which intensity of a certain return period is related to duration of rainfall event
> An IDF curve enables the hydrologists to develop hydrologic systems that consider worst-case scenarios of rainfall intensity and duration during a given interval of time
$>$ For instance, in urban watersheds, flooding may occur such that large volumes of water may not be handled by the storm water system appropriate values of precipitation intensities and frequencies should be considered in the design of the hydrologic systems

Cont....

* The use of IDF curve
- It is often used by entering with the duration and frequency to find the intensity. Or
- It is used to find the frequency for a measured storm event. The predicted frequency is determined by finding the intersection of the lines defined by the measured intensity and the storm duration.


IDF curves plotted on double logarithmic scale for Bahir Dar Station

## Mathematical Representation of IDF Curves

- The following equations can provide a reasonably accurate representation of an IDF curve:

$$
i=\left\{\begin{aligned}
\frac{a}{D+b} & \text { for } D \leqslant 2 \mathrm{hr} \\
c D^{d} & \text { for } D>2 \mathrm{hr}
\end{aligned}\right.
$$

Where
i is the rainfall intensity ( $\mathrm{cm} / \mathrm{hr}$ )
D is the duration (hr)and
$\mathrm{a}, \mathrm{b}, \mathrm{c}$, and d are fitting coefficients that vary with the frequency. Values for the coefficients are easily obtained by transforming the Equation to linear forms and evaluating the coefficients using any two points of the IDF curve by solving two simultaneous equations.

- How to transform the first equation in to linear equation

$$
\begin{aligned}
i & =\frac{a}{b+D} \\
\frac{1}{i} & =\frac{b+D}{a} \\
y & =f+g D
\end{aligned}
$$

Where
$\mathrm{y}=\frac{1}{i}$
$\mathrm{f}=\frac{b}{a}$ and
$\mathrm{g}=\frac{1}{a}$

- How to transform the second equation in to linear equation

$$
\begin{gathered}
i=c D^{d} \\
\log i=\log c+d \log D \\
y=h+d x
\end{gathered}
$$

Where
$y=\log i$,
$\mathrm{h}=\log \mathrm{c}$, and
$x=\log D$.

* The accuracy depends
$>$ on the reading accuracy of the two points selected and on the ability of Equation s to represent the shape of the IDF curve.
$>$ Greater accuracy can be achieved by using numerous points from the IDF curve and fitting the coefficients of the linear Equations using least squares


## Examples on IDF

From IDF curve of 2-yr frequency two Points are taken: $(i=4.1$ $\mathrm{cm} / \mathrm{hr}, \mathrm{D}=1 / 6 \mathrm{hr}$ ) and ( $\mathrm{i}=0.81, \mathrm{D}=2$ ). Develop the formula for the IDF curve.
solution

$$
\begin{aligned}
& 0.2439=f+g(0.1667) \\
& 1.2346=f+g(2)
\end{aligned}
$$

Solving Equations for f and g yields $\mathrm{f}=0.1538$ and $\mathrm{g}=$ 0.5404 . Solving for a and b yields the following:

$$
i=\frac{1.8505}{0.2847+D} \quad \text { for } D \leqslant 2 \mathrm{hr}
$$

- For durations greater than 2 hr , the following two points are taken from IDF curve, develop a formula for the IDF curve be $(\mathrm{i}=0.5, \mathrm{D}=4)$ and $(\mathrm{i}=0.3, \mathrm{D}=8)$.
- Solution
- Making natural-logarithm transformation yields $d=-0.7370$ and $\mathrm{h}=0.3286$. thus the equation becomes

$$
i=1,389 D^{-0.7370} \quad \text { for } D>2 \mathrm{hr}
$$

